Numerical Study on Measuring Electrical Resistivity through Casing in a Layered Medium

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Summary

A numerical analysis of Kaufman's work on electrical measurements through metal casing was applied to a layered medium. A surface integral equation approach was used to calculate the potentials in a finite-length borehole. It was assumed that the sources are located along the vertical axis and the inhomogeneities have axial symmetry. For this analysis, the casing and layers are modeled as inhomogeneities in the host medium. The simulation had all electrodes in contact with the inner surface of the casing.

The situation of the unknown casing conductance and single target layer was simulated. It appears that the discontinuity of the electric field corresponds to the boundaries of the layer and the rate of change of the field is proportional to the resistivity of the adjacent formation. To determine the formation resistivity, two sets of electrode configurations were used. One calculated the casing conductance and the other estimated the second derivative of the potential. A three-point measurement was used to approximate the second derivative. Using the expression from Kaufman's paper, the formation resistivity was obtained from a three-point measurement of the potential through a highly conductive casing. These calculated values compared closely to the resistivities of the layer model.

Introduction

In recent years, there has been an increased interest to measure formation resistivity through metal casing. Resistivity measurements obtained by this method can aid in characterizing existing reservoirs for effective recovery of oil and gas, as well as geothermal heat, without the cost and time of drilling new wells. The method is also useful to monitor changes in resistivity caused by subsurface processes such as injection or leakage of contaminants from a waste site, flooding operations for enhanced oil recovery, or extraction processes of geothermal production.

Several patents have recently been issued which describe methods and devices that are capable of measuring formation resistivity through casing, (Kaufman, 1989; Vail, 1989; Gard et al., 1989). Currently, it is only known that Vail has developed and tested such a device, called Through-Casing Resistivity Tool (TCRT).

Kaufman (1990) investigated the behavior of the potential and its derivatives for a borehole with casing based on models of an infinite-length conductive pipe in a homogeneous medium. He showed that the second vertical derivative of the potential only varies with the casing conductance and formation resistivity for receivers located in the "intermediate zone" from the source. From his analysis, he concluded that if the casing conductance, S_c is known, then the formation resistivity, ρ_f measurements taken at that depth by using the expression:

$$\rho_f = S_c^{-1} \frac{1}{\phi} \frac{d^2\phi}{dz^2} \tag{1}$$

where ϕ is the potential.

This study will focus on applying the expressions from Kaufman's paper for the formation resistivity from electrical measurements through casing in a layered medium. A surface integral equation approach is used to calculated the potentials in a finite-length borehole. In this numerical analysis, the borehole fluid, casing, and layer are modeled as inhomogeneities in the host or background medium. It is assumed that the sources are coaxial on the vertical axis and the inhomogeneities have axial symmetry so that the cylindrical coordinate system can be used. Due to the axisymmetry, the fields can be described by a radial, ρ and vertical, z component.

Integral Equation Formulation

Application of Green's theorem to Poisson's equation will result in an expression of the potential as the superposition of a single-layer and double-layer potential. By employing the boundary conditions, a Fredholm integral equation of the second kind is obtained.

$$\phi(\mathbf{r}) = \phi_0(\mathbf{r}) + C\int_S \phi(\mathbf{r}^*)\frac{\partial}{\partial n^*} g(\mathbf{r}, \mathbf{r}^*) ds^* \tag{2}$$

where $\mathbf{r}$ and $\mathbf{r}^*$ are the field point and the source density point on S, respectively.

This integral equation has the potential function $\phi(\mathbf{r})$ being expressed as the sum of the external source(s), $\phi_0(\mathbf{r})$ and the product of a double layer potential and a normalized conductivity contrast, $C = \Delta\sigma/\sigma_0$. The conductivity contrast is given by: $\Delta\sigma = \sigma_0 - \sigma_1$, where σ_0 and σ_1 are the background and disturbing body conductivities, respectively. The double layer potential which is distributed over the surface of the inhomogeneity contains the unknown boundary values, $\phi(\mathbf{r}^*)$ and the normal derivative of the Green's function. The axisymmetric half-space Green's function, used for this problem, is derived in Schenkel and Morrison (1990) and can be written as:

$$g(\mathbf{r}, \mathbf{r}^*) = \frac{1}{4\pi}\int_0^\infty J_0(\lambda\rho)J_0(\lambda\rho^*)\left[e^{-\lambda|z-z^*|} + e^{-\lambda(z-z^*)}\right]d\lambda \quad (3)$$

where $J_0(x)$ is the Bessel function of order zero and the primed and unprimed values are the locations of the source and field points, respectively.

To determine the unknown boundary potentials, let the field point approach the surface of the disturbing body. The surface integral which is improper must be evaluated at the singular point. This evaluation can be found is Brebbia et al. (1984). This results in an additional term to integral equation (2) for $\mathbf{r} = \mathbf{r}^*$.

$$\phi(\mathbf{r}) = \phi_0(\mathbf{r}) + C\left[\int_S \phi(\mathbf{r}^*)\frac{\partial}{\partial n^*}g(\mathbf{r}, \mathbf{r}^*)ds^* + \frac{1}{2}\phi(\mathbf{r}^*)\ \delta(\mathbf{r}, \mathbf{r}^*)\right] \quad (4)$$

where $\delta(\mathbf{r}, \mathbf{r}^*)$ equal one for $\mathbf{r} = \mathbf{r}^*$ and zero elsewhere.

The integral equation (4) is solved by the approximation method of point match over subsectional bases (see Harrington, 1968). This method involves the expansion of the unknown function into a series of weighting and basis functions at N discrete points on the region of interest. Each basis function exist only over a subsection in the region and the corresponding weighting function will only affect the approximation of the unknown function over that subsection. The integral over the region is then approximated as a summation of integrals over the subsections. By using a piecewise constant weighting function, equation (4) can be approximated by:

$$\phi(\mathbf{r}) = \phi_0(\mathbf{r}) + C\left[\sum_{n=1}^{N} K(\mathbf{r}, \mathbf{r}_n^*)\ \phi(\mathbf{r}_n^*) + \frac{1}{2}\phi(\mathbf{r}_n^*)\delta(\mathbf{r}, \mathbf{r}_n^*)\right] \quad (5)$$

where:

$$K(\mathbf{r}, \mathbf{r}_n^*) = \int_{S_n} \frac{\partial}{\partial n^*}g(\mathbf{r}, \mathbf{r}_n^*)ds^*$$

This linear form of the integral equation must now be satisfied at each N discrete points. As a result, a matrix equation is obtained and can be solved to determine the unknown basis functions. Once the basis functions are found, equation (5) is used to calculate the potential at the field point.

Results

Figure 1 illustrates the model used for this numerical analysis. The simplified model consists of a finite-length conductive casing filled with fluid embedded in a three-layer medium. For simplicity, the resistivity of 10 Ω·m was used for the top layer, bottom layer, and borehole fluid. The casing has a resistivity of 10^{-6} Ω·m and length of 100 m. The target layer was 3 m thick with its top located 49 m below the surface. To approximate a layer of infinite extent, the outer boundary of the layer placed at 5 km. The resistivity values for the target layer ranged from 1 Ω·m to 100 Ω·m. For the purpose of concentrating on the target layer, the cement which lines the casing was not included in the model. The equally spaced potential electrodes, M, N, and M', are straddled by two current electrodes, A and B. All electrodes are placed in contact with the casing. For models with single current source, electrode B is placed at "infinity", the remote position.

The calculated voltage difference for a lateral-log configuration is shown in Figure 2. These voltage differences, which are normalized by the potential electrode separation (MN = 0.5 m), represent an estimate of the electric fields on the casing and are proportional to the current leakage. The separation from the source to center potential array, AO, was 2.0 m. The resistivities of 1 Ω·m and 100 Ω·m were used for the target layer. The discontinuity of the curves corresponds to the change in resistivity. For the conductive layer, the increase rate of voltage drop is due to the increase current leakage in to the adjacent formation. A resistive formation has the opposite effect and a decrease rate of change is observed through the target layer.

A well-log of through casing measurements with unknown casing conductivity is simulated using the parameters in Figure 1. For an unknown casing conductance, two electrode configurations were used to obtain the formation resistivity by equation (1). In Figure 3, the top electrode configuration estimated the casing conductance and the bottom configuration was used to calculate the formation resistivity. To estimate the second derivative, three potential electrodes, M, M', and N, were needed. The current, I, is applied at the source electrodes, A and B, which are in close proximity to the potential electrodes. Due the large casing-formation conductivity contrast, the voltage across the electrode pair, MN or NM', is mainly dependent on the casing resistance between the pair. From the voltage and current strength, the casing conductance can be calculated. Moving the electrode B to the remote position, the current now must flow through the casing and the formation. The voltages differences, V_1 and V_2, at their respective electrode pair, MN and NM', will reflect both the casing and formation resistances. By subtracting the two voltages, a second derivative approximation is obtained.

With the estimate of the casing conductance and second derivative, the formation resistivity is calculated from equation (1). The numerical results for several resistivities of the target layer are illustrated in Figure 4. The distance between the potential and current electrodes was within the intermediate zone. In this zone, Kaufman indicated that the formation resistivity can be determined from the potential and its second derivative. For this analysis, the separation between current and center potential electrode, AN, was 2.25 m. The potential electrode spacings, MN and NM', were 1.0 m. From the plot, the calculated resistivity values are lower than that of the model for the homogeneous (no layer) and low resistivity (1 Ω·m to 5 Ω·m) layers. The discrepancy increases with decreasing resistivity, but decreases for increasing layer resistivity. The resolution of the layer boundaries is about 2.0 m which corresponds to the length used to approximate the second derivative of the potential.

Conclusion

The results obtained in the study appear to agree with Kaufman's investigation. An estimate of the formation resistivity can be obtain from the potential, its second derivative, and casing conductance by using equation (1). The vertical resolution of the layer boundaries seem to be dependent on th electrode spacing needed to approximate the second derivative.

The discrepancy of the calculated results and the model resistivity may be attributed to numerical errors, such as discretization of the model and approximating derivatives with differences. The discrepancy may also be the result of applying theory based on a homogeneous model to a more complex model (finite-length and layer). Additional analysis is needed to study the effects of variable casing conductance and the cement layer, as well as the response near the end of the casing.

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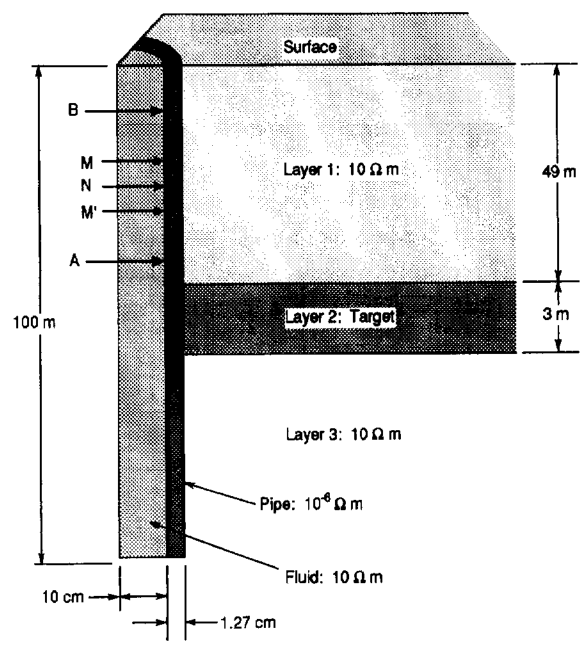

Figure 1: Model configuration and electrode array.

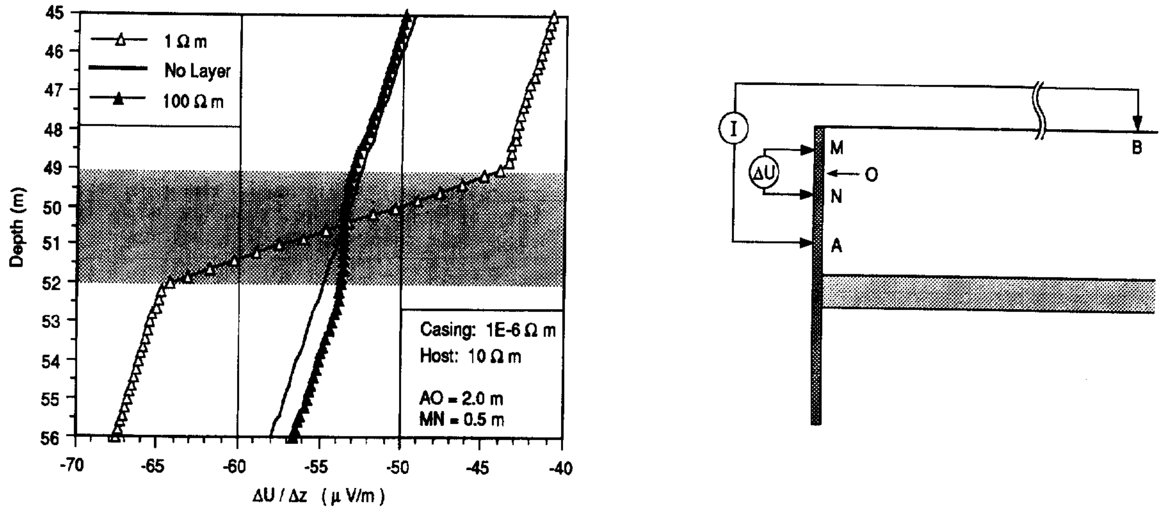

Figure 2: Electric field response with increasing depth (left) for a 1 Ω·m and 100 Ω·m layer (dark area). Electrode array (right) used for electric field calculation.

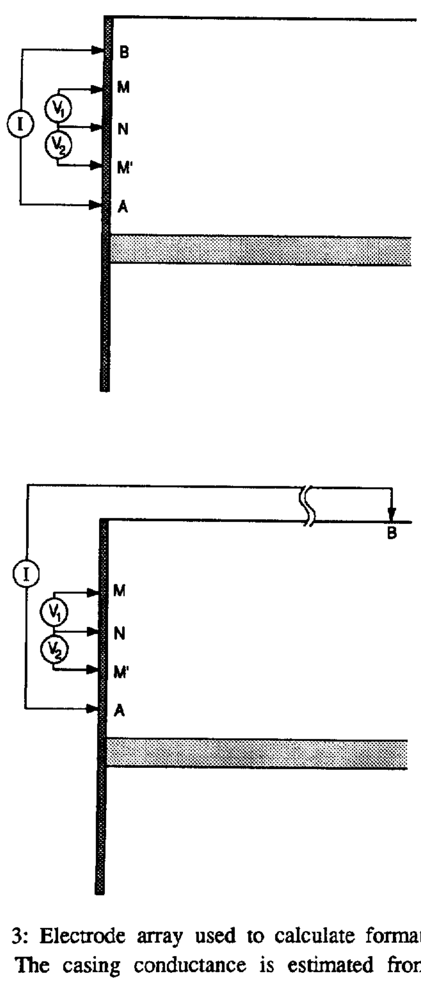

Figure 3: Electrode array used to calculate formation resistivity. The casing conductance is estimated from the top array and the second derivative of the potential is approximated with the bottom array.

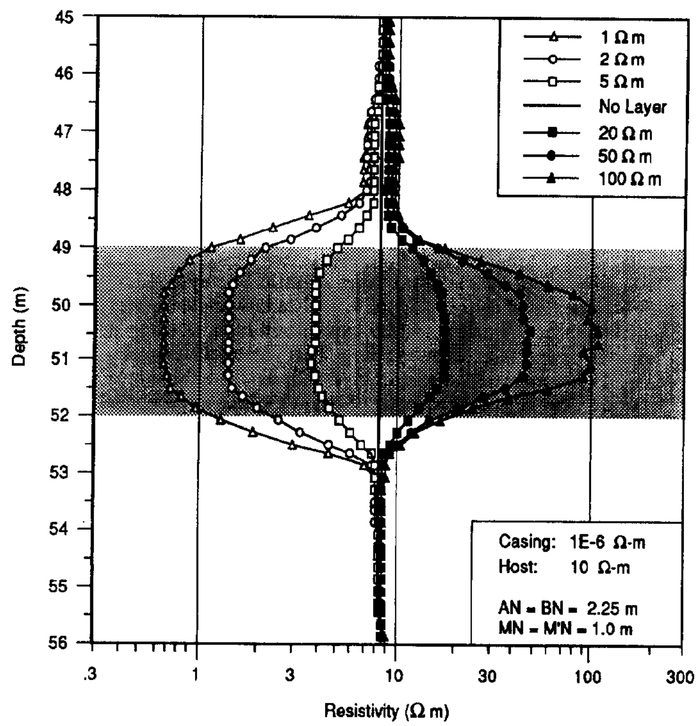

Figure 4: The resistivities calculated from the model of Figure 1 using equation (1). Several resistivities were used for the target layer (dark area).